Goldstino condensates and anti-brane instability

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Based on [2203.12636], G. Dall'agata, ME, F. Farakos, M. Morittu

Anti-branes and non-linear SUSY

Anti-branes are a common ingredient of string theory constructions used to break supersymmetry

De Sitter constructions, non-extremal BH microstates, ...

Kachru, et al '03, Balasubramanian et al '05, Bena et al '11 , etc

Positive contribution to the energy, without disrupting stability (?)

The supersymmetry breaking is spontaneous

Worldvolume contains goldstino sector. Non-linear realization of supersymmetry that can be described by constrained superfields

Bergshoeff et al '15, Vercnocke, Wrase '16

Coupling to supergravity is well understood at the EFT level

Lindstrom and Roček '79, Bergshoeff, et al '15, Bandos, et al '15

Volkov–Akulov model

A single Weyl fermion G, with action given by

Volkov, Akulov '73

$$\mathcal{L} = -f^2 + iG\sigma^m\partial_m\overline{G} - \frac{1}{4f^2}\overline{G}^2\partial^2 G^2 - \frac{1}{16f^6}G^2\overline{G}^2\partial^2 G^2\partial^2\overline{G}^2 \tag{1}$$

$\sqrt{f}$ is the SUSY-breaking scale.

Supersymmetry non-linearly realized

$$\delta_\epsilon G = -\sqrt{2}f\epsilon + \dots \tag{2}$$

Self-interaction terms required by supersymmetry.
May lead to composite states.

Nilpotent Chiral Multiplet

The V–A model can be written via a nilpotent chiral multiplet

Roček '78

$$X = \phi + \sqrt{2}\theta G + \theta^2 F \ , \ X^2 = 0 \implies \phi = \frac{G^2}{2F} \tag{3}$$

with

$$K = |X|^2 \ , \quad W = fX \tag{4}$$

In the presence of other field content, scalar potential given by the usual formula

$$V = K^{\bar{i}j}\overline{\partial_i W}\partial_j W \tag{5}$$

and subsequently setting $X| = \phi = 0$

Nilpotency Constraint via Lagrange Multiplier

Nilpotency constraint can also be imposed via a Lagrange multiplier chiral superfield

Casalbuoni et al '89, Komargodski, Seiberg '09

$$T = \tau + \sqrt{2}\theta\lambda + \theta^2 B \tag{6}$$

Kahler and superpotential become

$$K = |X|^2 \ , \quad W = fX + \frac{1}{2} T X^2 \tag{7}$$

At the component level, the τ and B equations of motion impose the nilpotency conditions $\phi = G^2/F$, $\phi^2 = 0$, while λ decouples.

Fermion Composite States

On-shell values of the scalars

$$\langle X \rangle \ \sim \ \left\langle \frac{G^2}{f} \right\rangle + \dots \ , \quad \langle T \rangle \ \sim \ \left\langle \frac{\partial^2 \overline{G}^2}{f^2} \right\rangle + \dots \tag{8}$$

T initially a Lagrange multiplier, but may acquire a kinetic term via quantum corrections.

Correct sign kinetic term for $T \implies$ composite states
(c.f. composite Higgs models, Nambu–Jona-Lasinio, etc)

Revealing Composite States

V–A action is a Wilsonian action defined at UV scale $\Lambda < \sqrt{f}$.

Find Wilsonian RG flow for the EFT in the X, T formulation.

Look for solutions to the RG flow, where the kinetic term of T vanishes at the original UV scale.

Near the UV point, small kinetic term implies strong coupling, so can't use weak coupling expansion.

Need a non-perturbative approach.

Exact Renormalization Group

Given a Wilsonian action $S[\Phi;\mu]$ with cutoff μ, the partition function

$$\begin{aligned}
\mathcal{Z}[\Phi] &= \int \mathcal{D}\Phi \; e^{S_{\text{reg.}}[\Phi;\mu]+S_{\text{int.}}[\Phi;\mu]} \\
S_{\text{reg.}}[\Phi;\mu] &= \int \frac{d^4k}{(2\pi)^4}\Phi^A(-k)C^{-1}_{AB}(k)\Phi^B(k) \\
S_{\text{int.}}[\Phi;\mu] &= \int \sum_\lambda g_\lambda(\mu) \prod_{\{A\}_\lambda} \left(\frac{d^4k_A}{(2\pi)^4} k_A^{n_A} \Phi^A(k_A) \right) \delta\Big(\sum_{\{A\}_\lambda} k_A \Big)
\end{aligned} \tag{9}$$

should be independent of μ

Exact Renormalization Group (ERG)

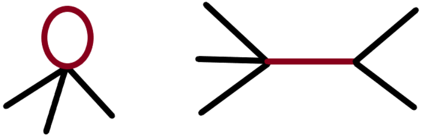
This leads to a condition of the form

Polchinski '84

$$\dot{S}_{\text{int.}} \equiv -\mu\partial_\mu S_{\text{int.}}$$
$$= \int \frac{d^4k}{(2\pi)^4}\tilde{C}^{AB}(k)\left(\frac{\delta^2 S_{\text{int.}}}{\delta\Phi^A(-k)\delta\Phi^B(k)} + \frac{\delta S_{\text{int.}}}{\delta\Phi^A(-k)}\frac{\delta S_{\text{int.}}}{\delta\Phi^B(k)}\right)$$

where $\tilde{C}^{AB}$ is related to C^{AB} in a prescribed way.

The terms have a diagrammatic interpretation



but this is not a small-coupling expansion!

Supersymmetric Local Potential Approximation (SLPA)

The ERGE are an infinite set of coupled ODE's. Need to truncate.

Can maintain supersymmetry by projecting onto interactions that are captured by Kahler and superpotential.

Kinetic terms are included in the flow, but not higher derivatives.

Anomalous dimensions are ignored (but can easily be incorporated).

Valid for small changes of μ, before truncated terms "backreact".

ERG for Volkov–Akulov

Consider the theory that includes newly generated terms

$$\begin{aligned} K &= \alpha|X|^2 + \beta|T|^2 + g\,|T|^2|X|^2 + \frac{1}{4}q\,|X|^4 \\ W &= fX + \frac{1}{2}TX^2 \end{aligned} \tag{10}$$

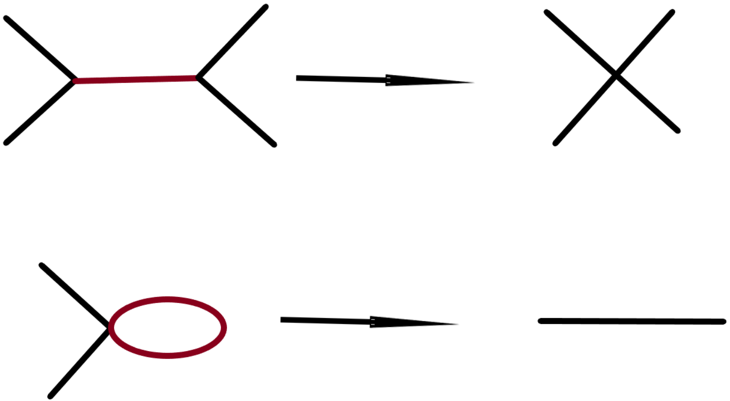
and separate K into a (regularized) propagator and interaction part

$$\begin{aligned} K_{\text{reg.}} &= c^{-1}|X|^2 + c^{-1}|T|^2 \\ K_{\text{int.}} &= (\alpha-1)|X|^2 + (\beta-1)|T|^2 + g|T|^2|X|^2 + \frac{1}{4}q|X|^4 \end{aligned} \tag{11}$$

with regulator function $c(p^2/\mu^2) = 1 + \sum c_n\; p^{2n}/\mu^{2n}$

ERG for Volkov–Akulov

Diagrammatically the calculation can be depicted as



ERG for Volkov–Akulov

The ERG equations for terms involving T are, in the SLPA,

$$\dot{\gamma} = -2\gamma - 2c_1 \ , \quad \dot{\beta} = -2N\gamma \tag{12}$$

where $\gamma = \mu^2 g$. Solve for boundary conditions

$$\beta\Big|_{t=0} = 0\,, \quad \gamma\Big|_{t=0} = 0\,, \tag{13}$$

with $t = \log(\Lambda/\mu)$, to find

$$\gamma = -c_1\left(1 - e^{-2t}\right) \sim -c_1 t$$

$$\beta = -c_1 N + 2c_1 N\left(t + \frac{1}{2}e^{-2t}\right) \sim 2c_1 N t^2$$

Where $c_1 < 0$ and $N < 0$ for typical monotonic regulator.

Consequences of the RG flow

T field becomes propagating, with positive kinetic term.
X no longer constrained. Positive 4-point couplings.

The mass matrix at $X = T = 0$ always has at least one negative eigenvalue, due to the TX^2 term in the superpotential.

$$m_\pm^2 = -\tilde{f}^2 \left[\left(\tilde{\gamma} + 4\tilde{\zeta}\right) \pm \sqrt{\frac{16\ \tilde{h}^2}{\tilde{f}^2} + \left(\tilde{\gamma} - 4\tilde{\zeta}\right)^2} \right] . \tag{14}$$

with $\tilde{f}, \tilde{\gamma}, \tilde{\zeta}, \tilde{h}$ all positive.

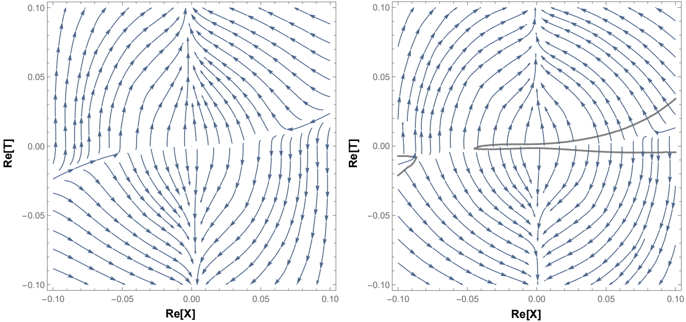
Qualitatively similar results hold if we naively insert the RG-evolved V–A model into KKLT, assuming small SUGRA corrections.

Resonates with existing literature on gravitino condensates!

Jasinschi, Smith '83-'84, Ellis, Mavromatos '13, Alexandre et al '13-'15

Consequences of the RG flow

Streamplots of the potential for $t = 0.1$ using $c(\hat{p}^2) = (1 - \hat{p}^2)\Theta(1 - \hat{p}^2)$. Pure V–A (left) and KKLT (right).



The composite states represented by T develop an expectation value. Within SLPA, the runaway is to infinity.

Outlook

Beyond SLPA: higher derivatives, anomalous dimension, regulator artifacts, etc.

Endpoint of instability? Positive energy? Supersymmetric?

Other constrained multiplets. Additional matter effects on RG flow.

RG flow in supergravity. (Beyond state of the art)

What does this imply about $\overline{D3}/O3$ system?
Does the anti-brane survive?

Worldsheet description? M-theory uplift? String Field Theory?

Summary and Conclusion

We used ERG equations to detect formation of fermion composite states in the Volkov–Akulov model

The model appears to be unstable toward goldstino condensation.

Potentially important consequences for anti-brane uplift models and anti-brane dynamics more generally

Instability of string dS vacua directly from 4D EFT perspective?

Many questions to explore on field theory and string theory fronts

Thank You!

Additional Slides: RG-evolved V–A model

Using the "optimized" regulator function $c(\hat{p}^2) = (1-\hat{p}^2)\Theta(1-\hat{p}^2)$, which gives $N = -\frac{1}{32\pi^2}$ and $c_1 = -1$. After canonically normalizing the fields, the Kahler and superpotential become

$$K \equiv |X|^2 + |T|^2 + \tilde{\zeta}|X|^4 + \tilde{\gamma}|X|^2|T|^2 =$$

$$= |X|^2 + |T|^2 + \frac{1}{4}\frac{(1-e^{-2t})}{\left[1-\frac{1}{16\pi^2}+\frac{1}{8\pi^2}(t+\frac{1}{2}e^{-2t})\right]^2}|X|^4 +$$

$$+ \frac{(1-e^{-2t})}{\left[1-\frac{1}{16\pi^2}+\frac{1}{8\pi^2}(t+\frac{1}{2}e^{-2t})\right]\left[-\frac{1}{32\pi^2}+\frac{1}{16\pi^2}(t+\frac{1}{2}e^{-2t})\right]}|X|^2|T|^2$$

$$W \equiv \tilde{f}X + \tilde{h}X^2T =$$

$$= \frac{e^{2t}}{\left[1-\frac{1}{16\pi^2}+\frac{1}{8\pi^2}(t+\frac{1}{2}e^{-2t})\right]^{1/2}}\xi_{UV}X +$$

$$+ \frac{1}{2}\frac{1}{\left[1-\frac{1}{16\pi^2}+\frac{1}{8\pi^2}(t+\frac{1}{2}e^{-2t})\right]\left[-\frac{1}{32\pi^2}+\frac{1}{16\pi^2}(t+\frac{1}{2}e^{-2t})\right]^{1/2}}X^2T$$

Additional Slides: Coupling to SUGRA and KKLT

As a first approximation, simply insert the Kahler and superpotentials into the SUGRA expressions

$$V = e^{e^{-2t}\frac{K}{P^2}} \left(K^{i\bar{j}} D_i W \, \overline{D_{\bar{j}} W} - 3e^{-2t} \frac{W \, \overline{W}}{P^2} \right)$$

$$D_i W = \partial_i W + e^{-2t} \frac{\partial_i K}{P^2} W \ , \qquad P = M_p / \Lambda$$

$$K = -3P^2 e^{2t} \log \left(\frac{S + \bar{S}}{P e^t} \right) + K_{VA.}$$

$$W = P^3 e^{3t} \left(W_0 + A e^{-\frac{aS}{Pe^t}} \right) + W_{VA.}$$

Corrections to the RG flow from SUGRA couplings are $1/P$ suppressed, and can't affect the tachyonic behavior.

Corrections to SLPA should show up before SUGRA corrections.

Additional Slides: RG flow of KKLT model

The energy at the critical point is a competition between a negative term $-3|W_0|^2$ and an "uplift" term f^2 coming from the fX term in the superpotential.

The uplift term decreases due to wavefunction renormalization of X, so the energy at the critical point flows down and becomes negative at $t \gtrsim 0.3$